

Anomalous Processes in Effective Chiral Theories at High Temperatures

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We discuss chiral theories of constituent quarks interacting with bosons at high temperatures. In the chirally symmetric phase we demonstrate by applying functional methods the presence of effective anomalous couplings for e.g. $\pi\sigma \rightarrow 2\gamma$, $\gamma \rightarrow \pi\pi\pi\sigma$, $KK \rightarrow \pi\pi\pi\sigma$, etc., as they have recently been discussed by Pisarski.

I. INTRODUCTION

In recent papers [1,2] Pisarski pointed out that although the chiral (abelian as well as nonabelian) anomaly [3,4] in terms of fundamental fields is temperature independent [5,6] the manifestation of the anomaly, however, in terms of effective fields changes with temperature. When considering e.g. $\pi^0 \rightarrow 2\gamma$ the observation in [1,2] is: "In a hot, chirally symmetric phase, π^0 doesn't go into 2γ , but $\pi^0\sigma$ does"! This statement indeed contradicts results which have been obtained previously [7,8].

In [1,2] the effective anomalous couplings for $\pi^0 \rightarrow 2\gamma$ ($\pi^0\sigma \rightarrow 2\gamma$) are found in the framework of the constituent quark model [4,9] by calculating the contribution from the Feynman one-loop triangle (box) diagrams at temperature $T = 0$ and at nonzero, high T , respectively.

In this note we attempt a different approach in order to confirm Pisarski's interesting result. As in [1,2] we start from the linear sigma model with constituent quarks interacting with bosons [10,11]. In order to include the effect of the axial anomaly on the bosons we transform the basis of right- and left-handed quarks following ref. [12], but in the situation of the chirally symmetric phase at high T [13]. At $T = 0$ and in the spontaneously broken phase this procedure allows a direct calculation of the Wess-Zumino-Witten action [14] for the Goldstone bosons, after applying functional methods for the evaluation of the fermion determinant in connection with path integrals [15].

In Sec. II we define the effective lagrangian and describe the way of performing the chirally symmetric limit at high temperature. Sec. III is devoted to the zeta-regularization used to evaluate the fermion determinant in the symmetric case. In Secs. IV and V we discuss the effective anomalous low-energy electromagnetic and hadronic couplings as a result of the chiral anomaly at high T .

II. EFFECTIVE CHIRAL LAGRANGIANS

We consider the $SU(2)_L \otimes SU(2)_R$ lagrangian for N_C coloured right- and left-handed quarks parametrized in the linear form (" Σ - basis"),

$$\mathcal{L} = \bar{\psi}_L i\partial^\mu \psi_L + \bar{\psi}_R i\partial^\mu \psi_R - g(\bar{\psi}_L \Sigma \psi_R + \bar{\psi}_R \Sigma^\dagger \psi_L) + \mathcal{L}_{boson}, \quad (1)$$

with the $SU(2)$ matrix $\Sigma = \sigma + i\vec{\tau} \cdot \vec{\pi}$, where we closely follow the notation used in [10]. In the following the explicit form of the boson lagrangian \mathcal{L}_{boson} is not needed [10,11]. Although in the strictly symmetric phase the constituent quark mass m has to vanish, we nevertheless introduce explicitly a breaking term

$$\mathcal{L}_m = -m\bar{\psi}\psi. \quad (2)$$

We treat m as an explicit regularization parameter, which is finally removed in the symmetric phase by performing the limit $m \rightarrow 0$. However, we do not break the chiral symmetry spontaneously by the standard redefinition of the scalar field σ .

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Following [12] we change the quark fields by

$$\psi_L' \equiv \xi^\dagger \psi_L , \quad \psi_R' \equiv \xi \psi_R , \quad (3)$$

(" ξ - basis") defining a unitary matrix ξ in such a way that the quark-pion coupling in Eq. (1) becomes replaced by a derivative coupling (as a consequence the physical predictions change in the low-energy limit, e.g. for $\pi^0 \rightarrow 2\gamma$, implying "anomalous inequivalence" as discussed for the case $T = 0$ in [12]).

In the symmetric phase we define for nonvanishing m

$$\xi^2 \equiv \frac{1 + g/m\Sigma}{(1 + g/m\Sigma)^{\frac{1}{2}} (1 + g/m\Sigma^\dagger)^{\frac{1}{2}}} . \quad (4)$$

Expanding in terms of the boson fields $(\sigma, \vec{\pi})$ the matrix ξ is equivalent (up to quadratic terms) to

$$\xi \simeq \exp\left[\frac{i\vec{\tau} \cdot \vec{\pi}}{2(m/g + \sigma)}\right] . \quad (5)$$

This matrix replaces the transformation matrix in the broken phase given by

$$\xi = \exp\left[\frac{i\vec{\tau} \cdot \vec{\pi}}{2F_\pi}\right] , \quad (6)$$

where F_π is the pion decay constant. In the chirally broken phase the transformation (6) may be obtained from (5) by first shifting $\sigma \rightarrow <\sigma> + \sigma \hat{=} F_\pi + \sigma$ and then performing the well defined limit $m \rightarrow 0$, while the dynamical σ field becomes heavy and is therefore assumed to decouple. However, approaching the symmetric phase, where $<\sigma>$ and F_π vanish, the σ field does not decouple and the mass parameter has to be kept as $m \neq 0$.

The transformation Eq. (3) yields

$$\mathcal{L} = \overline{\psi}'(i\cancel{D} - m)\psi' + \mathcal{L}_{boson} + O(g\sigma\overline{\psi}'\psi') , \quad (7)$$

with

$$D_\mu = \partial_\mu + i\overline{V}_\mu + i\overline{A}_\mu\gamma_5 , \quad (8)$$

where the currents [10] are defined by

$$\overline{V}_\mu = \frac{-i}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) , \quad \overline{A}_\mu = \frac{-i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) . \quad (9)$$

Explicitly, for infinitesimal transformations ξ the axial current \overline{A}_μ when expressed in terms of the boson fields becomes

$$\overline{A}_\mu = \frac{1}{2} \frac{1}{(m/g + \sigma)^2} \left[\left(\frac{m}{g} + \sigma \right) \vec{\tau} \cdot \partial_\mu \vec{\pi} - \vec{\tau} \cdot \vec{\pi} \partial_\mu \sigma \right] . \quad (10)$$

Keeping fixed $m \neq 0$ and expanding in the fields gives

$$\overline{A}_\mu \simeq \frac{1}{2} \left(\frac{g}{m} \right)^2 \left[\left(\frac{m}{g} - \sigma \right) \vec{\tau} \cdot \partial_\mu \vec{\pi} - \vec{\tau} \cdot \vec{\pi} \partial_\mu \sigma \right] + \dots , \quad (11)$$

where the dots indicate higher than quadratic terms in the $(\sigma, \vec{\pi})$ fields. It is important to notice that because of the factor $(g/m)^2$ the current \overline{A}_μ becomes singular for $m \rightarrow 0$, and as a consequence the symmetric limit is not immediately obtained.

Although the lagrangian remains invariant under the transformation, Eq. (3), the jacobian is not unity, as it is well known. Thus, in order to obtain equivalent low-energy physics in both representations, i.e. in the " Σ " as well as in the " ξ - basis", we have to calculate the fermion determinant for the transformation (3) - at $T = 0$ leading to the Wess-Zumino-Witten functional [14] - at high temperature, and finally we have to perform the chirally symmetric limit for $m \rightarrow 0$.

III. ZETA- REGULARIZATION AT HIGH TEMPERATURE

The jacobian for the transformation (3) of the quark fields is determined by standard steps. Here we closely follow the discussion in Chapters III, VII and in Appendix B of [10]. First one defines infinitesimal transformations by introducing a continuous parameter t , $0 \leq t \leq 1$, and extending the transformation of Eq. (5),

$$\xi \rightarrow \xi_t \equiv \exp\left[\frac{it\vec{\gamma} \cdot \vec{\pi}}{2(m/g + \sigma)}\right] \equiv \exp[i\tau\vec{\pi}] . \quad (12)$$

The effective action Γ , i.e. the Wess-Zumino-Witten action [14] in the broken phase, is defined by

$$\Gamma \equiv 2 \int_0^1 dt \operatorname{tr}(\bar{\pi}\gamma_5) \equiv \int d^4x \mathcal{L}_{odd} , \quad (13)$$

where the lagrangian \mathcal{L}_{odd} contains all the interactions in the odd-intrinsic-parity sector of the chiral model under consideration. In the following we only consider local terms as given by a Taylor expansion representation in t of Γ and \mathcal{L}_{odd} , respectively.

To regularize the trace in Eq. (13) we apply the zeta function method [16] combined with the expansion for the heat kernel $H(x, \tau)$ [10]. We start from

$$\operatorname{tr}(\bar{\pi}(x)\gamma_5) = N_C \operatorname{tr}' \int d^4x < x | \bar{\pi}(x)\gamma_5 | x > \Rightarrow N_C \int d^4x \operatorname{tr}' (\bar{\pi}(x)\gamma_5 \zeta_s(x)) , \quad (14)$$

(tr' indicates the Dirac and flavour trace) and introduce

$$\zeta_s(x) \equiv \frac{1}{\Gamma(s)} \int_0^\infty d\tau \tau^{s-1} H(x, \tau) , \quad (15)$$

with the heat kernel for massive fermions

$$H(x, \tau) \equiv < x | \exp[-\tau(D_\mu D^\mu + m^2)] | x > , \quad (16)$$

and the derivative D_μ given by Eq. (8).

At $T = 0$, in the broken phase, the jacobian (14) requires the regularization, Eq. (15), in order to suppress the contributions of the ultraviolet modes of the operator D_μ [15]. This is achieved by the ζ -function regularization starting with a convergent expression $\zeta_s(x)$ for $s > 0$, which is then analytically continued to $s = 0$. At high T , i.e. in the chirally symmetric phase, we propose to continue in the variable s in such a way that a non-trivial limit for \mathcal{L}_{odd} with $m \rightarrow 0$ is finally obtained.

A few technical steps have to be performed and described:

- the Dirac operator is defined by

$$D_\mu D^\mu + m^2 = d_\mu d^\mu + \sigma(x) + m^2 , \quad d_\mu = \partial_\mu + i\bar{V}_\mu(x) + \sigma_{\mu\nu} \bar{A}_\nu(x)\gamma_5 , \quad (17)$$

and $\sigma(x)$ as defined in [10]. For the following it is crucial that only even powers of the quark mass m are present in this operator [15];

- at high T it is convenient to continue first to euclidean coordinates and momenta ($p_E \equiv (\vec{p}, p_4)$);
- introducing a complete set of energy (Matsubara) and momentum eigenstates one obtaines the expansion

$$\begin{aligned} H(x, \tau) \simeq & T \sum \int \frac{d^3p}{(2\pi)^3} \exp[-\tau(p_E^2 + m^2)] \left[\frac{\tau^2}{2} (d \cdot d - \sigma)(d \cdot d - \sigma) \right. \\ & + \frac{4\tau^3}{3!} [p_E \cdot d \ p_E \cdot d (\sigma - d \cdot d) + p_E \cdot d (\sigma - d \cdot d) p_E \cdot d + (\sigma - d \cdot d) p_E \cdot d \ p_E \cdot d] \\ & \left. + \frac{16\tau^4}{4!} [p_E \cdot d \ p_E \cdot d \ p_E \cdot d \ p_E \cdot d] + \dots \right] . \end{aligned} \quad (18)$$

Here we drop i) terms $O(\tau^0, \tau^1)$, which give vanishing contributions after taking the Dirac trace in Eq. (13), and ii) terms, which either do not contribute at $T = 0$, or which lead to interactions in higher orders of the boson fields at high T . Σ sums the fermionic frequencies $p_4 = 2\pi T(n + 1/2)$ with integer n ;

- at $T = 0$ the integration over momentum is performed using the replacement

$$p_E^\mu p_E^\nu \rightarrow p_E^2 \delta^{\mu\nu}/4 , \quad (19)$$

and related ones (cf. [10]). At $T \neq 0$ one has to distinguish p_4 and \vec{p} components in the expression for $H(x, \tau)$, Eq. (18). We have checked, however, that the final results, summarized in Eqs. (27, 33), are identically derived by just using this ‘covariant’ prescription. To simplify the presentation we therefore choose to continue using Eq. (19) in the following;

- in order to evaluate the Matsubara frequency sum we first perform the τ -integration in Eq. (15) and then introduce the following expressions (cf. Appendix in [17])

$$T \Sigma \int \frac{d^3 p}{(2\pi)^3} \frac{1}{(p_E^2 + m^2)^{s+\beta}} \equiv \frac{(T^2)^{2-s-\beta}}{(2\pi)^{2s+2\beta-3}} I(y^2; s + \beta, \frac{1}{2}) , \quad (20)$$

where $y = m/(2\pi T)$ and the function I are dimensionless quantities. For $s > 0$ the function I has the following series expansion [17], in the following required for $\beta = 2, 3, \dots$,

$$\begin{aligned} I(y^2; s + \beta, \frac{1}{2}) &= \frac{1}{4\pi^{3/2}} \frac{1}{\Gamma(s + \beta)} \Sigma_0^\infty \frac{(-)^k}{k!} \Gamma(k + s + \beta - 3/2) \\ &\times \zeta(2k + 2s + 2\beta - 3, \frac{1}{2}) y^{2k} , \end{aligned} \quad (21)$$

where the Hurwitz zeta-function is related by $\zeta(z, \frac{1}{2}) \equiv (2^z - 1)\zeta(z)$ to the Riemann function $\zeta(z)$ [18]. It is crucial to realize that only even powers in (m/T) are present in Eqs. (20,21);

- taking care of the proper dimension of τ in Eq. (18) we multiply $\zeta_s(x)$ by T^{2s} ; furthermore we observe that the regularization prescription Eq. (15) still allows for an additional normalization factor $N(s)$, however, with the constraint $N(s=0)=1$; below we argue that

$$N(s) = \frac{(2\pi)^{2s} \Gamma(1/2 - s) \zeta(1 - 2s, \frac{1}{2})}{\Gamma(3/2) \Gamma(-s)} \quad (22)$$

for $s \leq 0$.

After performing these steps the function $\zeta_s(x)$ can be written as (in Minkowski metric)

$$\begin{aligned} \zeta_s(x) &\simeq \frac{\pi N(s)}{(2\pi)^{2s+2} \Gamma(s)} \\ &\times \left((d \cdot d - \sigma)^2 \Gamma(s+2) I(y^2; s+2, \frac{1}{2}) \right. \\ &+ \frac{1}{3}[2d \cdot d(d \cdot d - \sigma) + d_\mu(d \cdot d - \sigma)d^\mu] \Gamma(s+3) [y^2 I(y^2; s+3, \frac{1}{2}) - I(y^2; s+2, \frac{1}{2})] \\ &+ \frac{1}{18}[(d \cdot d)^2 + d_\mu d_\nu d^\mu d^\nu + d_\mu(d \cdot d)d^\mu] \Gamma(s+4) \\ &\times \left. [y^4 I(y^2; s+4, \frac{1}{2}) - 2y^2 I(y^2; s+3, \frac{1}{2}) + I(y^2; s+2, \frac{1}{2})] + \dots \right) . \end{aligned} \quad (23)$$

In case the functions d_μ and σ in Eq. (17) are entirely expressed in terms of fundamental fields, which occurs when studying the divergence of the axial currents in connection with the abelian as well as nonabelian anomalies, including their T -dependence, the limit $m \rightarrow 0$ is trivial: it amounts to take only the first term in the series of Eqs. (21,23). The s -dependence of $\zeta_s(x)$ becomes proportional to $\zeta(2s+1, \frac{1}{2})/\Gamma(s)$, which is only nonvanishing when $s=0$, due to the single pole of the Hurwitz function $\zeta(z, \frac{1}{2})$ at $z=1$. As a result [10] we obtain

$$\zeta_s(x) \xrightarrow{s \rightarrow 0} \frac{1}{(4\pi)^2} \left(\frac{1}{2}\sigma^2 + \frac{1}{12}[d_\mu, d_\nu][d^\mu, d^\nu] + \frac{1}{6}[d_\mu[d^\mu, \sigma]] \right) , \quad (24)$$

independent of T . Since the divergence of the anomalous (abelian and nonabelian) axial current is directly determined by $tr'(\gamma_5\zeta_{s=0}(x))$, where (d_μ, σ) are expressed in terms of gauge fields [3,4,10], the anomalies do not depend on the temperature [5,6]: the well known result is here reproduced with the regularization prescription under discussion.

However, for the case of the effective theories, one first has to collect the terms present in $\zeta_s(x)$ with different powers in m , i.e. in y^2 , in order to finally perform the chirally symmetric limit. The nonvanishing and regular terms to be found in this limit give rise to anomalous interactions in the odd-intrinsic-parity sector of the effective chiral theories at high temperatures. These low-energy couplings are, however, different from the ones obtained in the broken phase as pointed out in [1,2].

IV. ANOMALOUS ELECTROMAGNETIC COUPLINGS

We now derive the anomalous coupling in leading order for $\pi^0\sigma \rightarrow 2\gamma$ in the chirally symmetric phase. We introduce the electromagnetic interaction by replacing (cf. Eq. (17)): $d_\mu \rightarrow d_\mu + ieQA_\mu$, where A_μ is the photon field and $Q = 1/2$ ($1/3 + \tau_3$) (for $SU(2)$). The thermal bath is constituted by the π^0 and σ fields, to which external photons are coupled, i.e. we assume that the photons are not thermalized due to their small electromagnetic coupling. In this respect we differ from [2], where the photons are kept in thermal equilibrium with π^0 and σ .

From the discussion in Sec. III we look for nonvanishing contributions to

$$\mathcal{L}_T^{2\gamma} = \lim_{m \rightarrow 0} 2N_C \int_0^1 dt \, tr' \left(\frac{\tau_3 \pi^0}{2(m/g + \sigma)} \gamma_5 \zeta_s(x)|_{2\gamma} \right), \quad (25)$$

where the two-photon part $\zeta_s(x)|_{2\gamma}$ is provided by terms $O(\sigma^2)$ in Eq. (23) as $\sigma \sim \sigma_{\mu\nu}F^{\mu\nu}$ [10]. Expanding in the fields $(\sigma, \vec{\pi})$, e.g. $1/(m/g + \sigma) \simeq g/m - g^2/m^2\sigma$, it is seen, that the symmetric limit $m \rightarrow 0$ is no longer trivial in Eq. (25). However, regrouping terms in Eqs. (21,23) one finds an expansion with respect to $y^2 = m^2/(2\pi T)^2$ with coefficients proportional to

$$y^{2k} \frac{\zeta(2k+2s+1, \frac{1}{2})}{\Gamma(s)}, \quad k = 0, 1, 2, \dots . \quad (26)$$

Then by adjusting the value of s to $s = -1$ the term proportional to y^2 indeed leads to a finite nonvanishing value for $m^2 \rightarrow 0$. In addition all the other coefficients for $k \neq 1$ vanish due to the pole of $\Gamma(s)$ at $s = -1$. From Eqs. (23) and (24) we calculate

$$\zeta_{s=-1}(x) = -\zeta(3, \frac{1}{2}) y^2 \zeta_{s=0}(x), \quad (27)$$

using the definitions given in Eqs. (21,22).

The effective lagrangian for $\pi^0\sigma \rightarrow 2\gamma$ thus becomes

$$\mathcal{L}_T^{2\gamma} \simeq \frac{N_C e^2}{16\pi^2} \zeta(3, \frac{1}{2}) \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \lim_{m \rightarrow 0} \left(\frac{m}{2\pi T} \right)^2 Tr \left(\frac{\pi^0 \tau_3 Q^2}{2[m/g + \sigma]} \right) . \quad (28)$$

Keeping the term linear in the σ field the $m \rightarrow 0$ limit is defined,

$$\mathcal{L}_T^{2\gamma} = -\frac{7\zeta(3) g^2 \alpha N_C}{96\pi^3 T^2} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} (\pi^0 \sigma) . \quad (29)$$

We note that the effective photon coupling to neutral particles (π^0, σ) given in Eq. (29) is manifestly gauge invariant. By comparing the above coupling at high T with the one at $T = 0$ [4,14] we note the substitution

$$\frac{1}{F_\pi} \rightarrow \frac{7\zeta(3)g^2}{4\pi^2 T^2} \sigma, \quad (30)$$

which is in agreement with the derivation given in [1] based on evaluating triangle and box diagrams, respectively. Actually it is this relation which allows to fix the normalization factor $N(s)$ of Eq. (22) by the requirement:

$$T \Sigma \int \frac{d^3 p}{(2\pi)^3} \frac{1}{(p_E^2 + m^2)^{2-s}} \stackrel{T \gg m}{\sim} N(s) \left(\frac{T}{m} \right)^{2s} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p_E^2 + m^2)^{2-s}} . \quad (31)$$

In a corresponding fashion the anomalous couplings linear in $F_{\mu\nu}$ for e.g. $\gamma \rightarrow 3\pi$ σ may be derived, which turn out to be in leading order (keeping $s = -2$) proportional to $\zeta(5) g^4/T^4$ at high temperature.

Generalizing the result of Eq. (29) to the case of $SU(3)$ ($\vec{\pi}$ matrices are replaced by the Gell-Mann matrices $\vec{\lambda}$, the pion field $\vec{\pi}$ by the eight pseudoscalar bosons $\vec{\phi} = \pi, K, \eta_8$, correspondingly) allows to obtain the radiative two-photon couplings for the processes η (η') $\sigma \rightarrow 2\gamma$ in the chirally symmetric limit (besides the corresponding $T = 0$ normalization and the $\eta - \eta'$ mixing [10]). There are anomalous couplings between photons and mesons with more than one σ field, which, however, we do not consider here.

V. ANOMALOUS HADRONIC COUPLINGS

We now consider the anomalous couplings of interacting bosons at high temperature in the chirally symmetric phase. Following the expansion of the heat kernel, Eq. (23), these interactions are contained in the terms of $O(d^4)$, namely: $(d \cdot d)^2$, $d_\mu(d \cdot d)d_\mu$ and $d_\mu d_\nu d_\mu d_\nu$, respectively, where $d_\mu \sim \sigma_{\mu\nu} \bar{A}_\nu \gamma_5$ with \bar{A}_μ given by the $SU(3)$ version of Eq. (10).

It is useful to summarize shortly the $T = 0$ case: in the broken phase the simplest well known example of hadronic interactions is the one for $K^+ K^- \rightarrow \pi^+ \pi^- \pi^0$. After performing the Dirac trace (cf. Eq. (14)), and the Taylor expansion in the variable t , the leading term has the following structure [10]

$$\mathcal{L}_{T=0} \simeq \frac{8N_C}{12\pi^2} \int_0^1 dt t^4 \text{Tr} (\bar{\varphi} \epsilon^{\mu\nu\rho\sigma} \tilde{A}_\mu \tilde{A}_\nu \tilde{A}_\rho \tilde{A}_\sigma) , \quad (32)$$

where Tr denotes the $SU(3)$ flavour trace, with $\bar{\pi} \rightarrow \bar{\varphi} \equiv \vec{\lambda} \cdot \vec{\varphi}/2F_\pi$ and $\tilde{A}^\mu \equiv \partial^\mu \bar{\varphi}$ for $SU(3)$.

In order to describe $KK \rightarrow \pi\pi\pi\sigma$ interactions at $T \neq 0$ [1,2] in the symmetric phase we have to find the nonvanishing - finite - terms of Γ , Eq. (13), i.e. of $\zeta_s(x)$ in Eq. (23) in the limit $m \rightarrow 0$. Here the lowest power in $1/m^2$ from the product of $\bar{\varphi}$ and the fourth power in the axial current \bar{A}_μ of Eq. (10) present in the $O(d^4)$ terms is $1/m^6$. So by adjusting the value of s to $s = -3$ the term proportional to y^6 in (23) leads to a finite nonvanishing limit for $m \rightarrow 0$. Again all the other contributions vanish due to the pole of $\Gamma(s)$ at $s = -3$ (cf. Eq. (26)). We calculate

$$\zeta_{s=-3}(x) = \frac{15}{8} \zeta(7, \frac{1}{2}) y^6 \zeta_{s=0}(x) . \quad (33)$$

Expanding in t as for $T = 0$ $\zeta_{s=0}(x)$ is expressed in terms of the current \bar{A}_μ , Eq. (10), and we find the leading hadronic interaction linear in the σ field at high T given by

$$\begin{aligned} \mathcal{L}_T &\simeq \frac{127}{128} \frac{\zeta(7)}{\pi^2} \frac{N_C}{2\pi T} \left(\frac{g}{2\pi T} \right)^6 \\ &\times \text{Tr} (\vec{\lambda} \cdot \vec{\varphi}) \epsilon^{\mu\nu\rho\tau} \left\{ \left[\left(\sigma \partial_\mu (\vec{\lambda} \cdot \vec{\varphi}) - (\vec{\lambda} \cdot \vec{\varphi}) \partial_\mu \sigma \right) \partial_\nu (\vec{\lambda} \cdot \vec{\varphi}) \partial_\rho (\vec{\lambda} \cdot \vec{\varphi}) \partial_\tau (\vec{\lambda} \cdot \vec{\varphi}) \right. \right. \\ &+ \text{ cyclic permutations of } (\mu, \nu, \rho, \tau)] \\ &\left. \left. - 9\sigma \partial_\mu (\vec{\lambda} \cdot \vec{\varphi}) \partial_\nu (\vec{\lambda} \cdot \vec{\varphi}) \partial_\rho (\vec{\lambda} \cdot \vec{\varphi}) \partial_\tau (\vec{\lambda} \cdot \vec{\varphi}) \right\} . \right. \end{aligned} \quad (34)$$

The value of the zeta-function is $\zeta(7) = 1.00835$ [18]. This is the effective lagrangian at high T in the symmetric phase, which replaces the one of Eq. (32) at $T = 0$. \mathcal{L}_T shows a strong suppression with increasing temperature (for fixed coupling g).

Other anomalous interactions with more than one σ field may be worked out in a similar way; but again this requires the proper adjustment of the parameter s when performing the symmetric limit $m^2 \rightarrow 0$. At high T we do not find an universal \mathcal{L}_{odd} which can be given in a simple compact form, comparable to the Wess-Zumino-Witten action Γ_{WZW} at $T = 0$ [14].

In the spirit of effective lagrangians derived in the hard thermal loop expansion [19] Pisarski argues in [2] that local anomalous interactions as given by Eq. (34) are strictly valid only in the static limit [20]. The nonstatic case is expected to be more complicated than Eq. (34) because nonlocal contributions have to be included at high T .

VI. CONCLUSIONS

Finally, one may ask about phenomenological consequences, i.e. possible observable effects, of these high temperature anomalous couplings. An interesting case for testing Eq. (29) - and those related to the anomalous interactions of vector mesons at high T [1,2] - are photon- and dilepton measurements.

In this context we mention the production of low-mass lepton pairs in heavy ion collisions at high energies [21,22]. In order to interpret these data, especially for masses below and near the ρ meson mass, a detailed knowledge of the electromagnetic interactions of $\pi^0, \eta, \eta', \omega, \rho$, etc. in a high density hadronic environment (i.e. maybe in a chirally symmetric phase at high T) is important.

As a preliminary example based on the coupling of Eq. (29) we estimate the production rate of dileptons from the thermal production due to the process $\sigma\pi \rightarrow \gamma^*\gamma \rightarrow e^+e^- \gamma$ by using a few simplifying assumptions like e.g. vanishing π and σ masses, thermal equilibrium with Boltzmann distributions. In order to quote a temperature independent quantity we relate the resulting rate to the one calculated from the ρ dominated thermal process: $\pi\pi \rightarrow e^+e^-$ by evaluating both rates at the ρ peak, i.e. for $m_{e^+e^-} \simeq m_\rho$. The ratio is extremely small, it is of the order: $10^{-4} \alpha g^4 (\Gamma_\rho/m_\rho)^2$.

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- $$-5 \frac{127\zeta(7)g^6N_C}{256\pi^8T^6} \sigma \epsilon^{ijk} \text{Tr} \left(\frac{\vec{\lambda}}{2} \cdot \vec{\varphi} \right) \partial_0 \left(\frac{\vec{\lambda}}{2} \cdot \vec{\varphi} \right) \partial_i \left(\frac{\vec{\lambda}}{2} \cdot \vec{\varphi} \right) \partial_j \left(\frac{\vec{\lambda}}{2} \cdot \vec{\varphi} \right) \partial_k \left(\frac{\vec{\lambda}}{2} \cdot \vec{\varphi} \right) .$$
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